Slow Crack Growth in Glasses and Ceramics Under Residual and Applied Stresses

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The problem of slow crack growth under residual stresses and externally applied loads in plates is considered. Even though the technique developed to treat the problem is quite general, in the solution given it is assumed that the plate contains a surface crack and the residual stresses are compressive near and at the surfaces and tensile in the interior. The crack would start growing subcritically when the stress intensity factor exceeds a threshold value. Initially the crack faces near the plate surface would remain closed. A crack-contact problem would, therefore, have to be solved to calculate the stress intensity factor. Depending on the relative magnitudes of the residual and applied stresses and the threshold and critical stress intensity factors, the subcritically growing crack would either be arrested or become unstable. The problem is solved and examples showing the time to crack arrest or failure are discussed.

Introduction

Fracture related failures in many engineering materials and components may generally be studied in certain specific stages. The first stage is usually the formation or initiation of a "crack" in the material. This usually occurs at an imperfection or a location of stress concentration in the material and is generally enhanced by adverse environmental conditions and cyclic loading. The second stage consists of the "subcritical" growth of the crack under sustained or cyclic loading. The third and final stage of the process is the fracture or rupture of the component under a single application of the peak load.

For a given component if fracture is considered to be a possible mode of failure, in designing the component generally the final dimensions is also verified by using a fracture criterion. In most cases such a criterion is based on an assumed static peak load and a conjectured flaw size. On the other hand, if the loading is cyclic in nature or if the material is susceptible to environmental cracking, the subcritical crack growth may prove to be quite important in assessing the service life of the component. In this case the slow crack propagation under creep, fatigue, or fatigue/creep needs to be more carefully examined.

In the case of ceramics, glasses, and other brittle materials to determine the fracture load for a given flaw size or the critical flaw size for a given peak load, the so-called linear elastic fracture mechanics appears to provide an extremely effective tool. Here the fracture criterion is quite simple and consists of the comparison of the fracture toughness of the material (as represented by the critical stress intensity factor K_{IC} which is a measure of the material's resistance to fracture) with the maximum value of the stress intensity factor representing the magnitude of the applied loads and severity of part/crack geometry. Similarly, the subcritical crack growth rate in creep and fatigue can very effectively be correlated by using the stress intensity factor and the crack growth characteristics of the material [1-5]. Thus, for service life estimates it is generally sufficient to determine the stress intensity factor from the solution of the related crack problem and to characterize the material with regard to its crack growth properties and fracture toughness.

Aside from the experiments needed for material characterization, the procedure for service life estimate is rather straightforward and generally amounts to the calculation of the stress intensity factor at the active crack tip. Some difficulty may, however, be encountered in this procedure if the material is under residual stresses as well as applied loads. Most electronic packaging components are, of course, subjected to residual stresses. A sample life estimation study in these type of materials could, therefore, be quite useful. The problem considered in this paper is that of a glass plate under residual stresses and uniform static tension. The plate is assumed to contain a surface crack which grows subcritically. In the absence of the applied stress σ_p (Fig. 1) the problem was discussed in [6]. Some details of the crack-contact problem in plates with residual stresses were also considered in [7].

Crack Growth Model

The process of slow crack growth under sustained loading is variously known in ceramics community as static fatigue, delayed fracture, stress corrosion, or creep. The primary measure of the process is the crack velocity db/dt, b being the crack length. Macroscopically, the crack velocity V is known to correlate quite effectively with the stress intensity factor K, V generally being a monotonously increasing function of K. In the empirical V versus K relationship there are usually a

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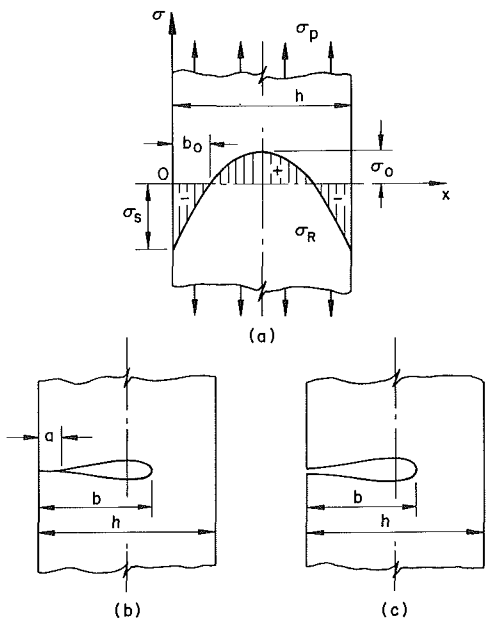

Fig. 1 Notation for the crack geometry and stresses

number of floating parameters which represent the properties of the material and are usually determined by curve-fitting to the experimental data. In addition to these internal parameters, V versus K relationship also depends on some external parameters, most notably the temperature and the concentration of the corrosive agents in the environment. As K, the primary crack driving force, approaches K_{IC}, the critical stress intensity factor, the crack velocity becomes unbounded. At the lower end of the loading scale the experiments indicate that one may define a threshold value of K (K_T) below which no crack growth may occur; that is, as K approaches K_T, V would tend to zero. These are the two important limiting cases that must be observed in the service life analysis.

The two most widely used empirical models for the crack velocity are [1], [2]

$$V = V_0(K/K_0)^n, \tag{1}$$

$$V = V_0\exp[(cK - E)/RT], \tag{2}$$

where V_0, K_0, and V_0, c are constants determined from the experimental data under given temperature and environmental conditions. The remaining constants E, R, and T are the apparent activation energy under zero stress, the gas constant and the absolute temperature, respectively. Even though the expressions (1) and (2) do not account for the asymptotic behavior of the data near K_T and K_{IC}, they can be made to represent by far the most significant portion of the subcritical crack growth process.

The Crack-Contact Problem

The problem under consideration is described in Fig. 1(a). For simplicity the surface crack problem in a flat plate is idealized by a plane strain problem. That is, in z direction the crack is assumed to be relatively long as compared to its depth b. Initially the plate is under self-equilibrating residual stresses $\sigma_{yy} = \sigma_R(x)$. It may also be subjected to uniform tension of magnitude σ_p. In the example given it will be assumed that the residual stresses are intentionally induced through tempering or ion exchange for the purpose of "strengthening" the plate. This is done by introducing compressive stresses to the part of the plate near and at its surfaces (Fig. 1(a)). However, the technique followed here is general and would apply to any self-equilibrating stress problem such as transient thermal stress problems and problems in bonded materials arising from thermal mismatch.

In the glass plate example considered as an application it is assumed that the stresses in the uncracked plate are (Fig. 1(a))

$$\sigma_R(x) = \sigma_0[1 - 3(1 - 2x/h)^2], \quad 0 < x < h, \tag{3}$$

$$\sigma_{yy}(x,y) = \sigma_R(x) + \sigma_p, \tag{4}$$

where σ_R and σ_p are respectively the residual and the remote stress, h is the plate thickness, and $\sigma_0 = \sigma_R(h/2)$ (or $\sigma_s = 2\sigma_0$) is a measure of the magnitude of σ_R.

The main problem here is essentially a crack problem shown in Fig. 1(b) or Fig. 1(c). Let b_0 correspond to the value of x where the sum of residual and applied stresses is zero, that is

$$\sigma_R(b_0) + \sigma_p = 0. \tag{5}$$

Note that for $0 \le p \le \sigma_s$ b_0 varies as $(1 - \sqrt{1/3})h/2 \ge b_0 \ge 0$. Thus, if a surface crack of depth b is introduced into the plate, for $b < b_0$ the crack is fully embedded in a compressive zone and therefore the crack surfaces would remain closed. However, if b is greater than b_0, then the crack tip region is in tensile zone, the crack would open and the corresponding stress intensity factor would have to be calculated. In this case, depending on the ratios σ_p/σ_s and b/h the crack may be opened either partially (Fig. 1(b)) or fully (Fig. 1(c)). It may be observed that regardless of the value of b/h, for $\sigma_p > \sigma_s$ the net section stress σ_{yy} is tensile everywhere and consequently the crack will always be open (as in Fig. 1(c)).

It is thus seen that the problem has an additional unknown a, namely the depth of the contact region near the surface. The analytical condition needed to determine a comes from the physical argument that at $x = a$ the crack surfaces close "smoothly"; that is at $x = a$ the crack tip has a cusp shape and, therefore, the corresponding stress intensity factor would be zero. The problem can then be solved by assuming that the plate contains an internal crack along ($y = 0$, $a < x < b$) and is subjected to combined stresses given by (4). The problem may be reduced to an integral equation of the form [8], [9]

$$\frac{1}{\pi}\int_a^b \left[\frac{1}{t-x} - \frac{1}{t+x} + \frac{6x}{(t+x)^2} - \frac{4x^2}{(t+x)^3} + k(x,t)\right] f(t)dt$$

$$= -\frac{1+\kappa}{4\mu}[\sigma_p + \sigma_R(x)], \quad (a < x < b) \tag{6}$$

Nomenclature

- a = depth of crack surface contact region
- b = depth of surface crack
- b_a = the value of crack depth at crack arrest
- b_0 = crack depth for zero combined stress
- b_i = the initial crack depth
- $b_1 = b_T$ = minimum crack depth for crack growth initiation
- $b_2 = b_c$ = critical crack depth (for unstable growth)
- h = plate thickness
- K_b = stress intensity factor
- K_T = threshold value of stress intensity factor for creep crack growth
- K_{IC} = critical stress intensity factor
- t = time
- T_a = time to crack arrest
- T_f = time to failure
- V = crack velocity
- σ_0 = residual stress in the midplane of the plate
- σ_s = residual stress on the surface of the plate
- σ_p = applied stress
- σ_R = the residual stress

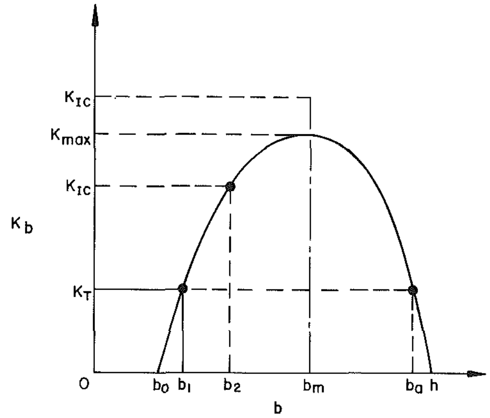

Fig. 2 Procedure for calculating the time to crack arrest or failure in a plate under residual stress only

where the unknown function f is the derivative of the crack surface displacement $v(x,0)$,

$$f(x) = \frac{\partial}{\partial x} v(x, +0), \quad (a<x<b) \tag{7}$$

and for $a>0$ must satisfy the following single-valuedness condition:

$$\int_a^b f(x)dx = 0. \tag{8}$$

μ and κ are the elastic constants (μ the shear modulus, $\kappa = 3-4\nu$ for plane strain and $\kappa=(3-\nu)/(1+\nu)$ for plane stress, ν being the Poisson's ratio). The bounded kernel $k(x,t)$ is a known function and may be found, for example, in [9].

For $a>0$ (Fig. 1) the solution of (6) has the form

$$f(x) = F_1(x)/\sqrt{(x-a)(b-x)}, \quad (a<x<b) \tag{9}$$

where $F_1(x)$ is bounded and is directly determined from (6). After determining $f(x)$ the stress intensity factors at the crack tips $x=b$ and $x=a$ may be defined and evaluated from

$$K_b = \lim_{x\to b+0} \sqrt{2\pi(x-b)}\sigma_{yy}(x,0) = -\frac{4\mu}{1+\kappa} \lim_{x\to b-0} \sqrt{2\pi(b-x)}f(x) = -\frac{4\mu}{1+\kappa} \frac{F_1(b)\sqrt{2\pi}}{\sqrt{b-a}}, \tag{10}$$

$$K_a = \lim_{x\to a-0} \sqrt{2\pi(a-x)}\sigma_{yy}(x,0) = \frac{4\mu}{1+\kappa} \lim_{x\to a+0} \sqrt{2\pi(x-a)}f(x) = \frac{4\mu}{1+\kappa} \frac{F_1(a)\sqrt{2\pi}}{\sqrt{b-a}}. \tag{11}$$

Thus the additional condition to evaluate a may be expressed as

$$K_a = 0 \text{ or } F_1(a) = 0. \tag{12}$$

If, on the other hand, σ_p/σ_s and b/h are sufficiently large so that $a=0$, then the integral equation (6) and the definition (9) are still valid (with $a=0$ and without the condition (8)) and (9) would have to be replaced by

$$f(x) = F_2(x)/\sqrt{b-x}, \quad (0 \le x < b) \tag{13}$$

and (10) becomes

$$K_b = -\frac{4\mu}{1+\kappa} F_2(b)\sqrt{2\pi}. \tag{14}$$

Time to Crack Arrest or to Failure

From the viewpoint of crack arrest or failure qualitatively the two cases of $\sigma_p = 0$ and $\sigma_p > 0$ was considered in [6]. If $\sigma_p = 0$ the crack closure distance a (Fig. 1(*b*)) is always positive and the stress intensity factor K_b is finite and has the form as shown in Fig. 2; that is K_b is zero for $0<b<b_0$, and for $b=h$, and positive for $b_0<b<h$. In this case the procedure for calculating the time to crack arrest or failure is described in Fig. 2. It is seen that if $K_b<K_T$ no crack growth would occur, if $K_b>K_T$ and $K_{\max}<K_{IC}$ the crack would grow and would be arrested at $b=b_a$, and if $K_b>K_T$ and $K_{\max}>K_{IC}$ the crack would grow and become unstable at $b=b_2$.

We note that for given stress magnitudes σ_s and σ_p (Fig. 1(*a*)) the stress intensity factor is a function of the crack depth b only. This function is obtained from the solution of the crack-contact problem described in the previous section and is assumed to be known as

$$K_b = \phi(b), \quad (b_0<b<h). \tag{15}$$

From the crack growth model we also know the relationship between the crack velocity and the stress intensity factor as, for example, expressed by (1) or (2). Thus, by substituting from (15) into (1) or (2) we obtain

$$V = \frac{db}{dt} = \psi(b) \tag{16}$$

where ψ is now a known (implicit) function of b. Thus, if $b=b_i$ is the crack length at $t=0$, the time as a function of crack length may be obtained by integrating (16)

$$t = \int_{b_i}^{b} \frac{db}{\psi(b)}. \tag{17}$$

Referring to (15), Fig. 2 and to the condition for crack growth $K_b>K_T$, it is seen that in (17) we must have $b_i \ge b_1$, where b_1 is obtained from

$$\phi(b_1) = K_T, \quad \phi'(b_1)>0. \tag{18}$$

In the absence of the applied stress σ_p and for $K_T<K_{\max}<K_{IC}$, the time to crack arrest T_a may then be obtained as (Fig. 2)

$$T_a = \int_{b_i}^{b_a} \frac{db}{\psi(b)}, \quad (b_i \ge b_1), \tag{19}$$

where the arrest length b_a of the crack is determined from (see (15) and Fig. 2)

$$\phi(b_a) = K_T, \quad \phi'(b_a)<0. \tag{20}$$

On the other hand, if $K_{\max}>K_{IC}$ and $b_i \ge b_1$ then the crack would eventually become unstable and the time to failure T_f may be calculated from (see Fig. 2)

$$T_f = \int_{b_i}^{b_2} \frac{db}{\psi(b)}, \tag{21}$$

where b_2 is defined by (Fig. 2)

$$\phi(b_2) = K_{IC}), \quad \phi'(b_2)>0. \tag{22}$$

For $\sigma_p=0$, by observing that K_b is directly proportional to the surface stress σ_s (Fig. 1(*a*)), from Fig. 2 one may also define two characteristic stress levels σ_T and σ_c as

$$K_{\max} = K_T \to \sigma_s = \sigma_T, \tag{23}$$

$$K_{\max} = K_{IC} \to \sigma_s = \sigma_c. \tag{24}$$

Thus, for $\sigma_s<\sigma_T$ the crack would be dormant (i.e., $T_a=\infty$) regardless of its initial size, for $\sigma_T<\sigma_s<\sigma_c$ the crack would grow and be arrested and for $\sigma_s>\sigma_c$ the crack would grow and become unstable.

If, in addition to the residual stress σ_R, the plate is subjected to a uniform tensile stress σ_p (Fig. 1(*a*)), the character of the problem and the results would change quite significantly. In this case for relatively high values of σ_p/σ_s and/or b/h the

Table 1 Slow crack growth and fracture properties of soda-lime-silica glass under ambient conditions (equation 2)

$\left.\begin{array}{l}\log V_0 = -1.08 \\ c = 0.188\end{array}\right\}$ for $K_b < 3.62 \times 10^5$ Nm$^{-3/2}$

$\left.\begin{array}{l}\log V_0 = 10.3 \\ c = 0.110\end{array}\right\}$ for $K_b > 3.62 \times 10^5$ Nm$^{-3/2}$

$E = 1.088 \times 10^5$ J mol^{-1} (activation energy)

$R = 8.32$ J mol^{-1}°C^{-1} (gas constant)

$T = 298$°K (absolute temperature)

$K_T = 2.49 \times 10^5$ Nm$^{-3/2}$

$K_{IC} = 7.49 \times 10^5$ Nm$^{-3/2}$

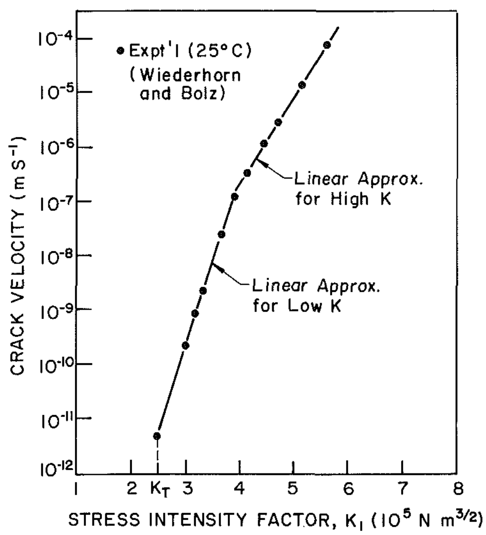

Fig. 3 Subcritical crack growth velocity in soda-lime-silica glass at 25°C

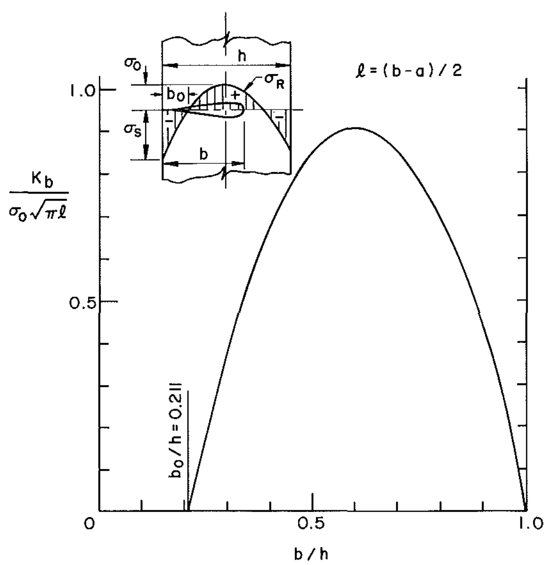

Fig. 4 Stress intensity factor in a plate containing a surface crack under residual stress only

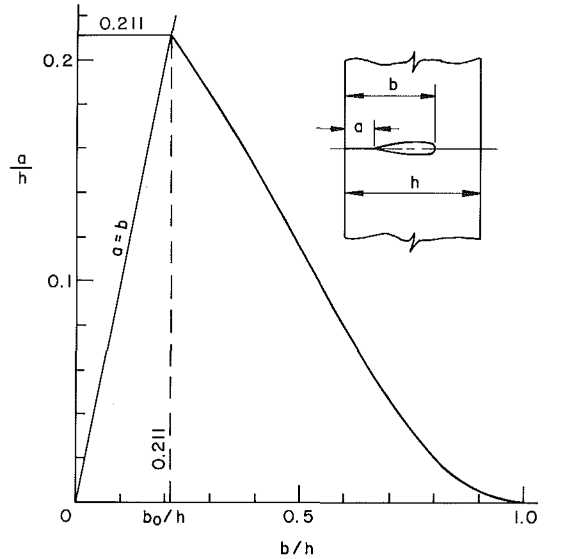

Fig. 5 The depth of the crack surface contact region in a plate under residual stress only

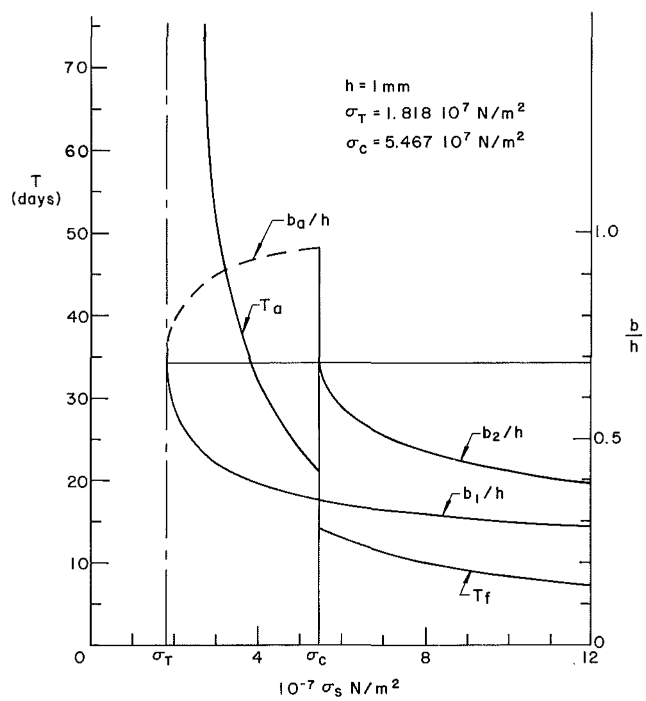

Fig. 6 Time to crack arrest T_a and to failure T_f in a 1 mm. thick glass plate under residual stress only

contact depth a would be zero and the crack would be completely open (Fig. 1(c)). The problem would then be a simple edge crack problem with given nonuniform crack surface tractions. It is also clear that since the net section stress in the uncracked plate has a net tensile component σ_p and, hence, is no longer self-equilibrating, as b approaches h the stress intensity factor K_b becomes unbounded. That is, theoretically, $K_{\max} = \infty$ and if K_b is a monotonously increasing function of b and if $b_i > b_1$ the crack would always become unstable. For this case the failure time T_f can again be obtained from (21) and (22). However, if K_b has a local maximum satisfying $K_T < K_{\max} < K_{IC}$ and a crack length b_a exists for which (20) is satisfied, then the crack arrest is possible and the arrest time may be calculated from (19).

In the simpler problem of a plate without any residual stresses and subjected to a uniform tensile stress σ_p the solution of the edge crack problem would give (*)

$$K_b = \phi(b) = \sigma_p\sqrt{\pi b}(1.1215 + 6.5200r^2 - 12.3877r^4 + 89.0554r^6 - 188.6080r^8 + 207.3870r^{10} - 32.0524r^{12}), \quad r = b/h, \tag{25}$$

(*) the relation (25) is obtained from the results given in [8] by a least square curve fitting and is valid in $0 < b/h < 0.85$.

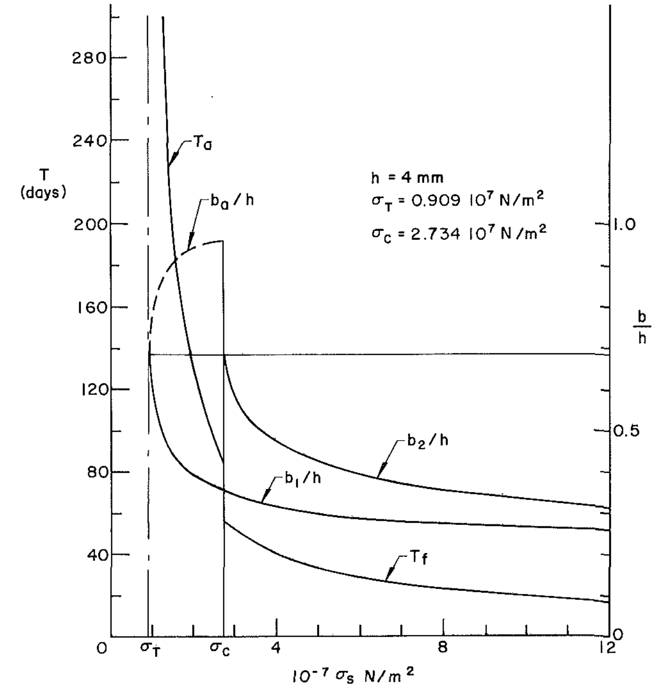

Fig. 7 Time to crack arrest T_a and to failure T_f in a 4 mm thick glass plate under residual stress only

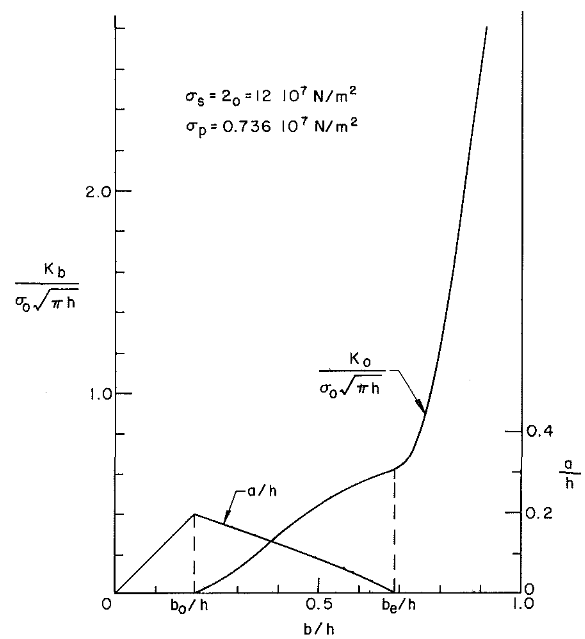

Fig. 8 The stress intensity factor K_b and the depth of the contact region a in a plate under residual and applied stresses ($\sigma_s = 12 \times 10^7$ N/m^2)

which is a monotonously increasing function of b. In this case, for a given σ_p the threshold value of the crack depth $b_T = b_1$ below which no crack growth would take place may approximately be obtained from

$$1.1215\sigma_p\sqrt{\pi b_T} \cong K_T. \qquad (26)$$

Results and Discussion

As an example the slow crack growth process in tempered soda-lime-silica glass of thicknesses varying from 1 to 8 mm under room temperature is considered. It is assumed that the residual stress distribution is parabolic and is given by (3), and the magnitude of the surface stress $\sigma_s = 2\sigma_0$ varies between 10^7

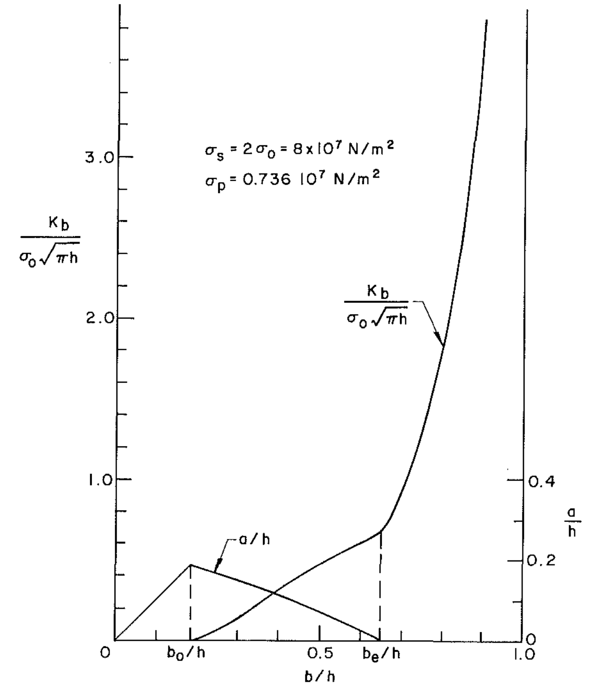

Fig. 9 The stress intensity factor K_b and the depth of the contact region a in a plate under residual and applied stresses ($\sigma_s = 8 \times 10^7$ N/m^2)

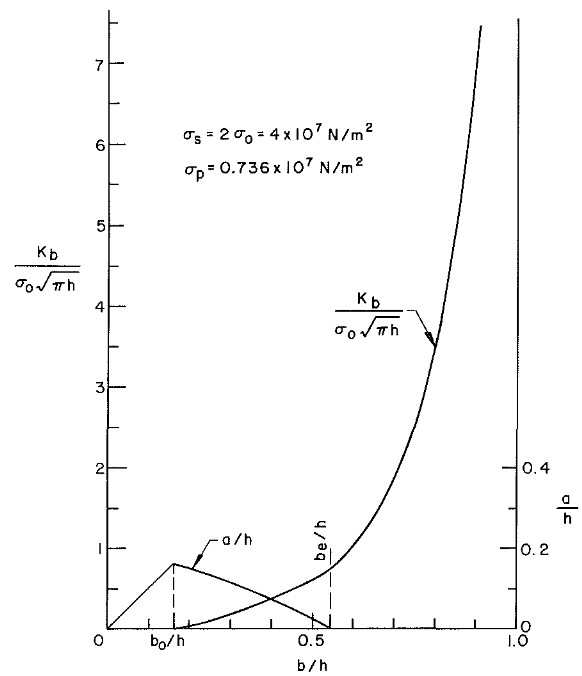

Fig. 10 The stress intensity factor K_b and the depth of the contact region a in a plate under residual and applied stresses ($\sigma_s = 4 \times 10^7$ N/m^2)

and 12×10^7 N/m^2. Table 1 shows the properties of the material [6]. The slow crack growth constants V_0 and c which appear in the creep model (2) are obtained from the experimental data shown in Fig. 3 [6], [1]. The data for the particular glass considered exhibits a sharp knee under moist environment. Consequently, log V versus K was approximated by a bilinear relationship as shown in Fig. 3 and in the Table. The case of $\sigma_p = 0$ was considered in [6] for a 2 mm thick plate. Here the results are given for 1 mm and 4 mm thick plates. A sample result for the normalized stress intensity factor is shown in Fig. 4. For $b < b_0$ the crack is closed and $K_b = 0$,

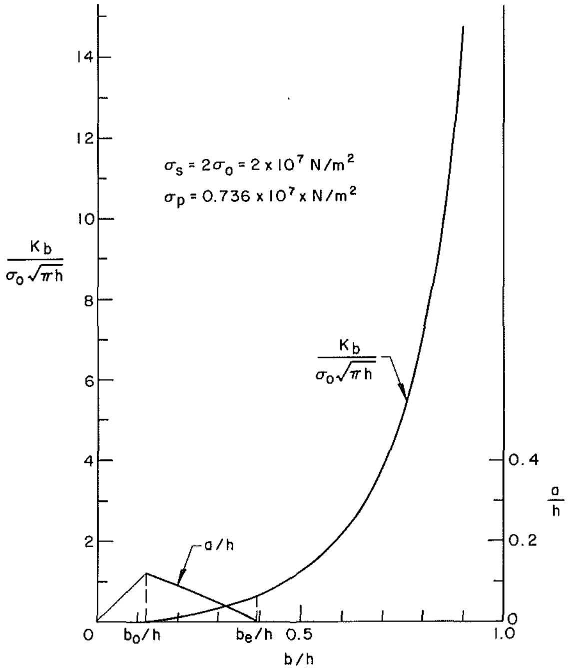

Fig. 11 The stress intensity factor K_b and the depth of the contact region a in a plate under residual and applied stresses ($\sigma_s = 2 \times 10^7$ N/m^2)

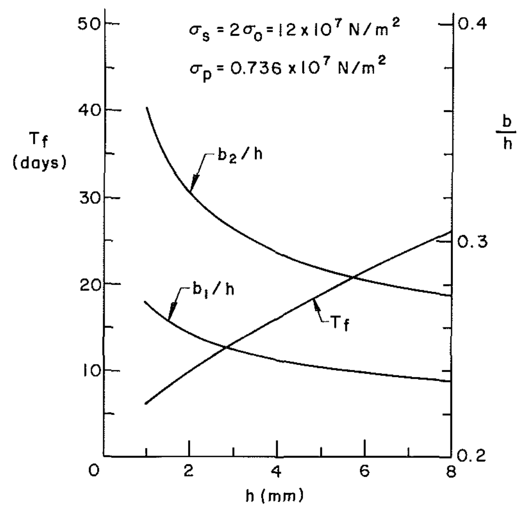

Fig. 12 Time to failure T_f, threshold crack depth b_1 and critical crack depth b_2 in a glass plate under residual and applied stresses ($\sigma_s = 12 \times 10^7$ N/m^2)

where b_0 is determined from (3) and (5) with $\sigma_p = 0$. For $b > b_0$ the crack has the shape shown in Fig. 1(*b*) and the calculated value of the contact region a is shown in Fig. 5. Note that when the crack is fully closed, i.e., for $b < b_0$, $a = b$ and as b increases from b_0 to h, a decreases and tends to zero.

For $\sigma_p = 0$ the main results showing the time to crack arrest or to failure as a function of the surface stress σ_s are shown in Fig. 6 and 7. In these and in all subsequent examples it is assumed that the initial crack length b_i used in calculating T_a and T_f in (19) and (21) is the same as b_1, the threshold value of the crack length. Figures 6 and 7 show the crack lengths b_1, b_a, and b_2 which are defined in Fig. 2 and are calculated from (18), (20) and (22), respectively. The importance of the characteristic stress levels σ_T and σ_c defined by (23) and (24) may now be seen from these figures. Note that as σ_s approaches σ_T, T_a tend to infinity, $K_{\max} \to K_T$ and, as a result, $b_1 \to b_m$ and $b_a \to b_m$, where b_m is the crack depth corresponding to $K_{\max}$ (Fig. 2). For $K_{\max} = K_{IC} - 0$ or $\sigma_s = \sigma_c - 0$, T_a is calculated from (19). For $\sigma_s > \sigma_c$ or $K_{\max} > K_{IC}$ (Fig. 2) the crack would become unstable and as $K_{\max} \to K_{IC} + 0$ or $\sigma_s \to \sigma_c + 0$ b_2 also tends to b_m. Theoretically, at $\sigma_s = \sigma_c$ the arrest and failure times are calculated from (see (19) and (21))

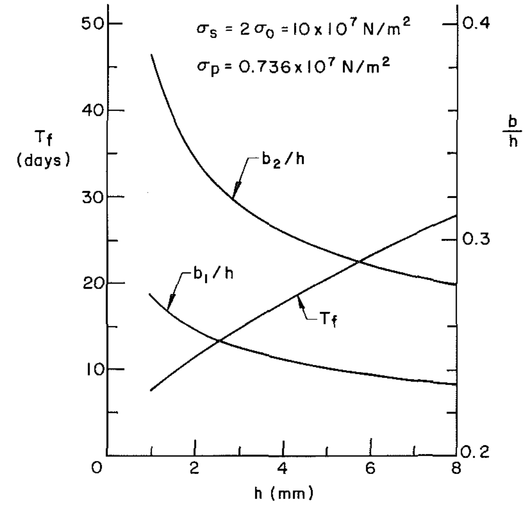

Fig. 13 Time to failure T_f, threshold crack depth b_1 and critical crack depth b_2 in a glass plate under residual and applied stresses ($\sigma_s = 10 \times 10^7$ N/m^2)

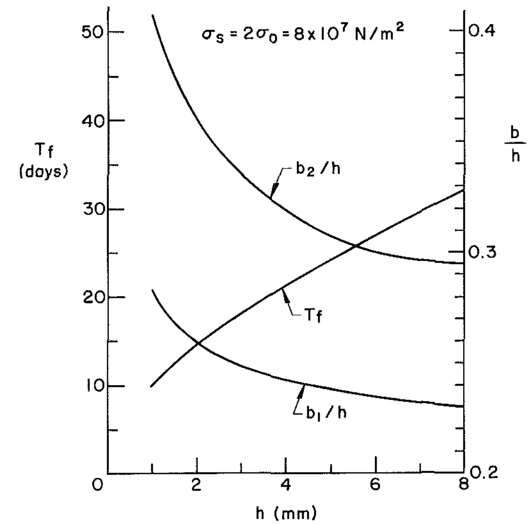

Fig. 14 Time to failure T_f, threshold crack depth b_1 and critical crack depth b_2 in a glass plate under residual and applied stresses ($\sigma_s = 8 \times 10^7$ N/m^2)

$$T_a = \int_{b_1}^{b_m} \frac{db}{\psi(b)} + \int_{b_m}^{b_a} \frac{db}{\psi(b)}, \tag{27}$$

$$T_f = \int_{b_1}^{b_m} \frac{db}{\psi(b)}, \tag{28}$$

hence the discontinuity in times shown in Figs. 6 and 7.

The results for plates under a uniform tensile stress σ_p as well as the residual stress σ_R are shown in Figs. 8–17. Here σ_R is again assumed to be parabolic and σ_p is constant ($\sigma_p = 0.736 \times 10^7$N/m^2). Figures 8–11 show the stress intensity factor K_b and the depth of the contact region a as functions of the crack depth for various values of σ_s. Again for $b < b_0$ the crack remains closed and $a = b$. b_0 shown in these figures is obtained from (5). For $b_0 < b < b_e$ a decreases and becomes zero at $b = b_e$. For $b > b_e$ the crack is completely open, that is $a = 0$. Note that b_0, b_e and a would all decrease as σ_p/σ_s increases.

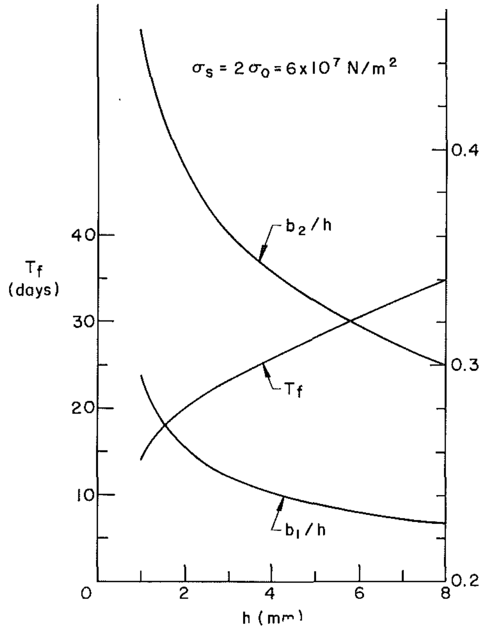

Fig. 15 Time to failure T_f, threshold crack depth b_1 and critical crack depth b_2 in a glass plate under residual and applied stresses ($\sigma_s = 6 \times 10^7$ N/m^2)

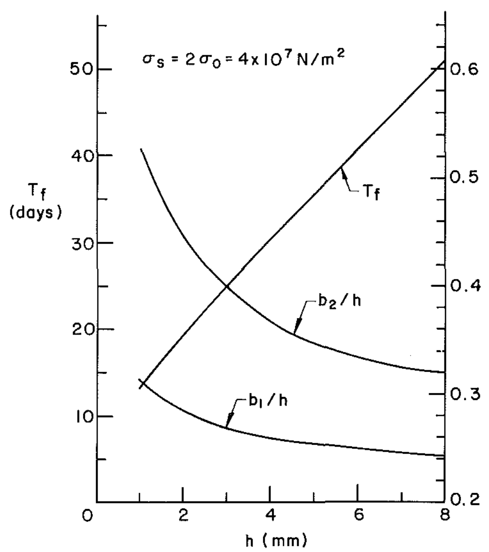

Fig. 16 Time to failure T_f, threshold crack depth b_1 and critical crack depth b_2 in a glass plate under residual and applied stresses ($\sigma_s = 4 \times 10^7$ N/m^2)

To simplify the subsequent analysis, in these figures K_b is normalized by using the constant plate thickness h rather than the crack length as the length parameter. It is seen that, at least for the values of σ_p/σ_s considered, K_b is a monotonously increasing function of b and hence no crack arrest is possible.

The failure times calculated from (21) are shown in Figs. 12–17 where it is again assumed that $b_i = b_1$ and b_2 is given by (22). The calculated values of b_1 and b_2 are also shown in the figures. It may be observed that for a given applied stress σ_p the time to failure T_f increases with increasing plate thickness h and with decreasing residual stress magnitude σ_s.

One factor which has not been taken into consideration in this study that could be important is the difference between the environmental conditions at the crack tip before and after

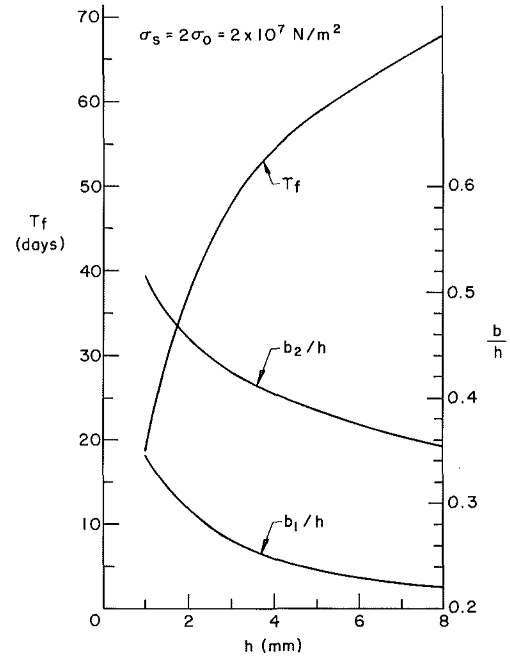

Fig. 17 Time to failure T_f, threshold crack depth b_1 and critical crack depth b_2 in a glass plate under residual and applied stresses ($\sigma_s = 2 \times 10^7$ N/m^2)

the contact length a becomes zero. For $a > 0$ it is quite possible that the crack tip may be isolated from the environment and such corrosive agents as moisture may not penetrate to the crack tip. Whereas for $b > b_e$ (Figs. 12–17) the crack is open and the crack tip is fully exposed to the environment. In this case the calculation of the stress intensity factor K_b would not be affected. The change would be in the material constants shown in Table 1 and for $b > b_e$ may easily be incorporated into the analysis.

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